SOLUTION BY C. E. HORNE, Westminster, Colorado.

Let $11x/y$ and y/x be the legs of the triangle, thus fulfilling one condition of the problem. Then

$$\frac{121x^2}{y^2}+\frac{y^2}{x^2}=\frac{121x^4+y^4}{x^2y^2}=\square.$$

This will be so when $121x^4+y^4=\square$. Let $\sqrt{121x^4+y^4}=11x^2+zy^2$. Then

$$x=y\sqrt{\frac{1-z^2}{22z}}$$

and

$$\frac{x}{y}=\sqrt{\frac{1-z^2}{22z}},$$

will be rational when

$$\sqrt{\frac{1-z^2}{22z}}=\square=\text{say } t^2.$$

Solving for z, we have $z=-11t^2\pm\sqrt{121t^4+1}$. When $t=3/70$ $z=-50/49$ or $49/50$. Hence, $x/y=3/70$; $11x/y=33/70$ and $y/x=70/3$. The hypotenuse $=4901/210$.

NOTE.—Having found one value of t which makes $\sqrt{121t^4+1}$, rational, we may find other values by making use of Euler's method, in his *Elements of Algebra*, third edition, revised by Hewlett, Chapter IX, p. 374.—EDITOR.

225. Proposed by W. DE W. CAIRNS, Oberlin College.

L'Intermédiaire for June, 1914, contains the following problem: "If we write the terms of the arithmetic series 1, 5, 9, 13, 17, 21, 25, 29, 33, $\cdots$ as follows:

$$\begin{array}{ccccccc} 1 \\ 5 & 9 & 13 \\ 17 & 21 & 25 & 29 & 33 \\ 37 & 41 & 45 & 49 & 53 & 57 & 61 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot, \end{array}$$

it is seen that the sum of the terms of each line is a cube, and that these are the cubes of the successive odd integers. How is this shown?"

It is here proposed not only to prove this, but to generalize the theorem as suggested, using, however, the simpler (and better known) case which includes all of the successive integers:

$$\begin{array}{cccc} 1 \\ 3 & 5 \\ 7 & 9 & 11 \\ 13 & 15 & 17 & 19 \\ \cdot & \cdot & \cdot & \cdot\,. \end{array}$$

SOLUTION BY THOS. E. MASON, Purdue University.

Theorem. Form the arithmetic series $1+2kn$, $n=0, 1, 2, 3, \cdots$, where k is any positive integer which remains constant for a given series. Arrange this

series in rows, 1 in the first row, and each row after the first containing k more terms than the preceding. Then the sum of the terms in any row is the cube of the number of terms in that row.

From the way in which the series is formed, the value of n for any term will be the same as the number of terms preceding it in the series. The number of terms in the rows will form an arithmetic series with first term 1 and difference k. Making use of these facts, we obtain the following:

The number of terms in the rth row is $1 + (r - 1)k$.

The value of n for the first term in the rth row is the sum of $r - 1$ terms of the arithmetic series with first term 1 and difference k, that is,

$$\frac{r-1}{2}[2 + (r - 2)k].$$

Making use of this value of n, we have for the first term in the rth row the value $1 + k(r - 1)[2 + (r - 2)k]$. The sum of the terms in the rth row will be the sum of $1 + (r - 1)k$ terms of the arithmetic series with first term $1 + k(r - 1)[2 + (r - 2)k]$ and difference $2k$, that is,

$$\frac{1 + (r-1)k}{2}\{2[1 + k(r - 1)\{2 + (r - 2)k\}] + k(r - 1)2k\} = \{1 + (r - 1)k\}^3.$$

This proves the theorem.

The two arrangements of the problem can be obtained by making $k = 2$ and $k = 1$, respectively.

Also solved by R. M. Mathews, Elijah Swift, Harmon L. Slobin, and S. A. Joffe.

QUESTIONS AND DISCUSSIONS.

Edited by U. G. Mitchell, University of Kansas.

At the time of making up copy for this issue further replies are desired to questions numbered 4, 8, 12, 13, 16, 20, 23, 24, 25 and 26.

NEW QUESTIONS.

27. A certain college wishes to offer twelve hours of mathematics beyond the usual courses in analytical geometry and differential and integral calculus. Considering only the needs of students intending to specialize in pure mathematics, what courses should make up the twelve hours offered?

28. Is it possible to obtain $\int \cos \theta^2 d\theta$ without expanding $\int \cos \theta^2$? If it is not, can some interesting properties of this integral be determined by treating it as a special function?

DISCUSSIONS.

RELATING TO ADJUSTABLE CALENDARS.

By Irwin Roman, Chicago, Ill.

So far as the writer has been able to learn, all perpetual or adjustable calendars are arranged so as to present the first day of the month as the first day of the week.